# Beaching angles and evolution of stack geometry for thickened tailings: A review

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## Abstract

*The reliable prediction or management of beach slope and stack geometry is integral to realization of the potential benefits of thickened tailings technology. Stack geometry not only controls the storage capacity for a given footprint, but also strongly influences post-deposition geotechnical and geoenvironmental performance. Unfortunately, accurate prediction in design has proved difficult. However, there has been significant improvement in understanding the behaviour of tailings after they exit the pipe, and a number of methods for beach slope prediction have been proposed and developed. This paper reviews a selection of these methods, and examines their applicability to different deposition scenarios. . Some recommendations are made to assist engineers and operators to achieve a given stack geometry.*

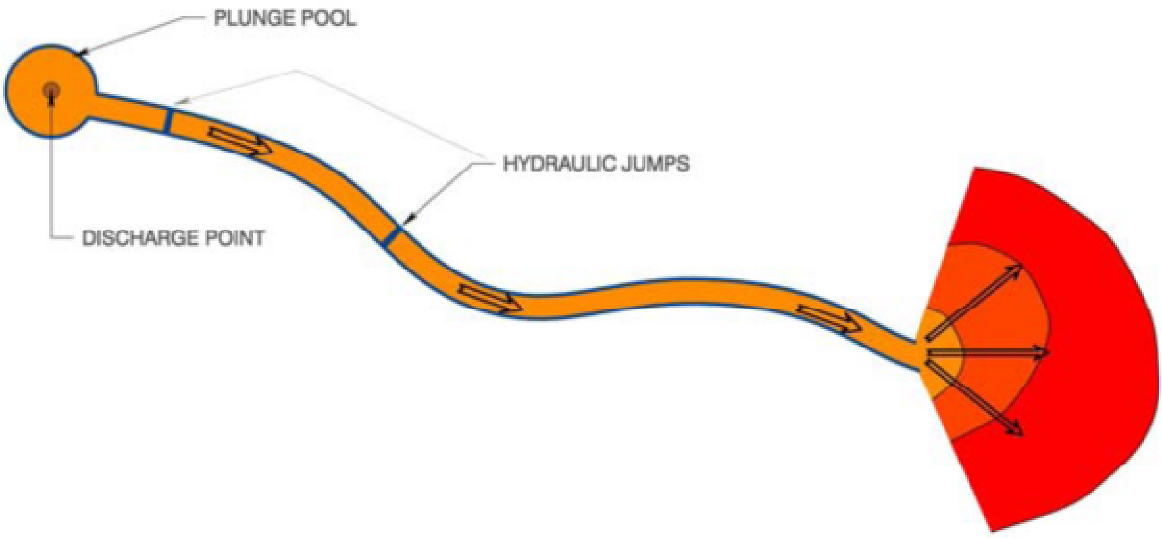
## 1 Introduction

Reliability of beach slope prediction and control of stack evolution in thickened tailings deposits remains a challenge that affects the credibility of thickened tailings technology. The overall slope angle is critical to stack design and estimation of storage capacity of a given impoundment. Too gentle of a slope will enlarge impoundment footprints or require dam raises, whereas too steep of a slope promotes erosion and decreases geotechnical stability of the stack. Aspects of stack geometry that may influence engineering performance other than the equilibrium beach slope include i) the evolving stack angle, ii) run out, and, iii) individual layer thicknesses (where deposition is cycled between a number of points). All these parameters strongly influence the geotechnical and geoenvironmental performance of the stack. In central deposition, the evolving slope angle influences runoff and lateral drainage, which contribute to desiccation. For cyclic deposition, the thickness of individual layers controls the rate of desiccation, and this thickness is determined by the flow rate and rheology of the tailings as they flow down the beach. Desiccation contributes to strength but excess desiccation is a concern in sulphidic tailings due to the possibility of oxidation and acid rock drainage (ARD) generation (Fisseha et al. 2010).

While the behaviour of thickened or paste tailings in the pipeline have been studied to a significant degree (Pullum et al. 2006; Sofra 2006; Nguyen and Boger, 1998), deposition geometry has received less attention. Many studies have focused on charactering thickened tailings geometry using a single angle (Kwak et al., 2005; Sofra and Boger, 2002) measured at the laboratory scale. However, it is observed that the average slope of deposits in the field are typically substantially less than slopes measured in the laboratory (Oxenford and Lord 2006; Engman et al., 2004). This was partially thought to be attributed to shear thinning occurring during transport (For example, Houman et al., 2007). However, field observations have shown that the behaviour of thickened tailings deposits is dependent on scale. Smaller or younger deposits are often characterized by relatively uniform spreading flow and convex beach slope profiles, while in older or larger stacks tailings will flow in narrow channels and the beach slope profile may be concave (Williams and Meynink 1986). Simms (2007) proposed, based on non-Newtonian flow theory that had been previously applied to mud and lava flows (Liu and Mei, 1989), that even deposits in the "spreading" stage are not characterized by a unique slope, and that the overall slope of the deposit is a function the size of the flow. Therefore, an average value of slope measured in a flume tests cannot be directly extrapolated to the field. Henriquez and Simms (2009) verified the applicability of lubrication theory to describe thickened tailings flows for a gold tailings at the laboratory, and observed the scale-dependency of the overall deposition angle of a single layer deposits. Gawu and Fourie (2010) applied equations derived from a geotechnical perspective and also noted the same scale dependency. Gawu and Fourie (2010) also investigated errors induced by sidewall friction in flume tests.

For prediction the equilibrium beach profile for stacks where channel flow dominates, some researchers have used the concept of equilibrium between erosion and sedimentary deposition (Fitton et al. 2008, Pirouz and Williams 2007). Others have pursued an approach based upon dissipation of kinetic energy of the tailings stream as it travels down the slope of the stack (McPhail 1995, 2008). The variety of theories probably reflects different possible states of the tailings as they flow: laminar or turbulent, supercritical or subcritical, spreading or channelized flow. For example, tailings may be observed to exit the pipe as a supercritical flow, which undergoes a hydraulic jump close to the deposition point, and subsequently converts into a spreading subcritical flow. On the same stack but later in the deposition process, the flow does not undergo a hydraulic jump, and rather forms a narrow channel, only spreading out near the bottom of the stack (Figure 1).

**(a)**




**(b)**

**Figure 1 Picture and schematic of initial channel flow at the deposition point, subsequently changing to spreading flow down slope (Schematic by MPA Williams, 2010)**

This paper presents in some detail three predictive methods that have shown some success in field comparisons. The methods are compared, and their relative applicability to different deposition strategies is discussed. The paper concludes with some recommendations to practitioners concerning deposition strategies to regulate stack geometry. Figure 2 is presented to give some sense to the reader of the important parameters and phenomena that can affect stack evolution and beach slope prediction. The reader will see that some predictive methods account for some of these behaviours more than others. For example, lubrication theory considers that flows are dominated by viscous forces and gravity, and inertia is ignored, while in other methods inertia is of prominent consideration.

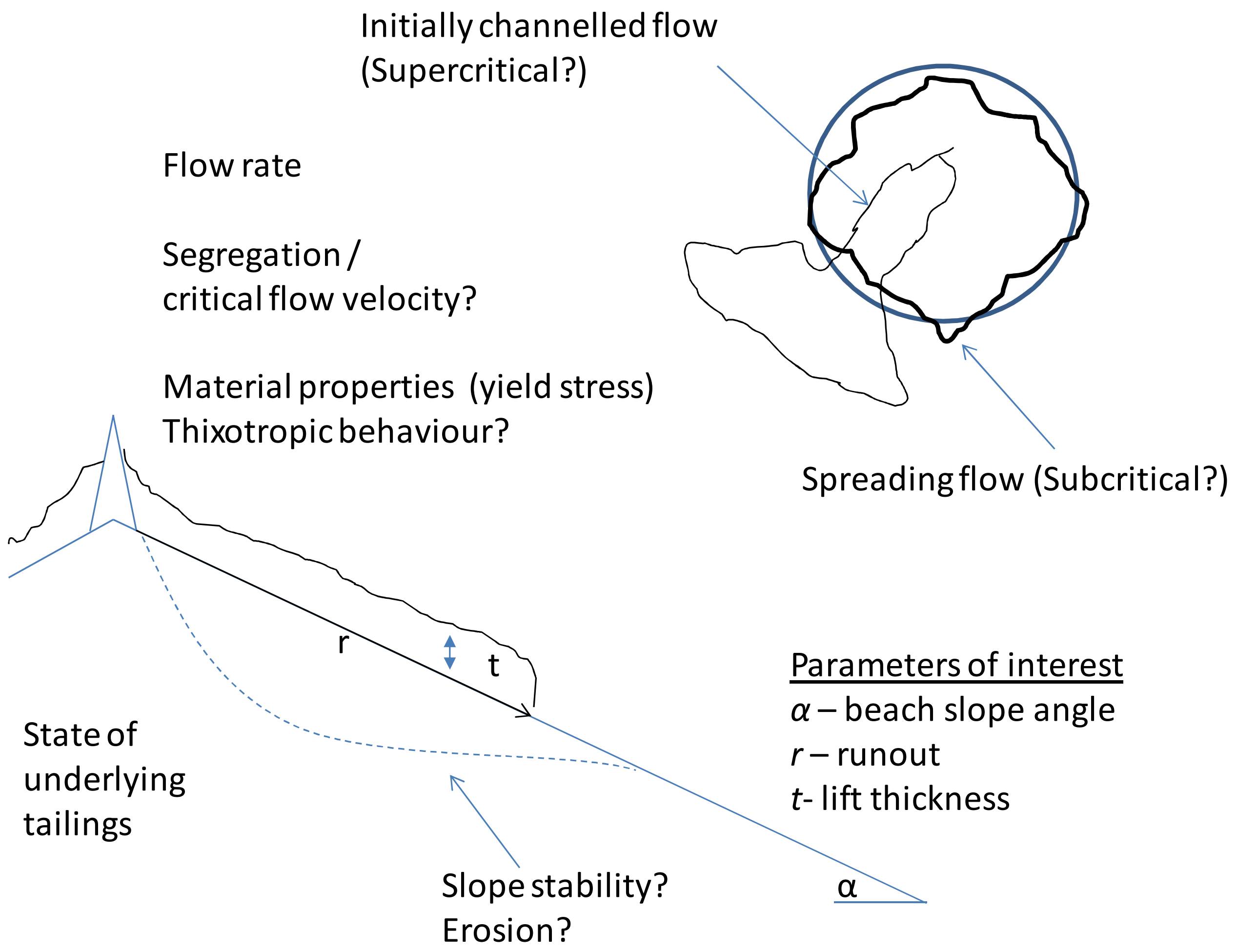



**Figure 2 Tailings and deposition properties and relevant phenomena pertinent to control of stack geometry**

## Method 1: Fitton Beach Slope Method

The following model has been largely developed by Tim Fitton (Fitton et al. 2009, Fitton 2007, Fitton et al. 2006 a and b). The model integrates non-Newtonian open channel flow and sediment transport. It is assumed that the maximum beach slope corresponds to the slope of a channelized tailings flow, in which the tendency for erosion and sedimentation reaches equilibrium. This equilibrium is characterized by the minimum transport velocity for the tailings, which can be determined experimentally, or using the following equations:

For segregating slurry, the Wasp (1977) equation is recommended:

$$V_c = 3.8{C_v}^{1/4}\left(\frac{d}{4R_H}\right)^{1/6}\left(\frac{8gR_H\,(\rho_s-\rho_l)}{\rho_l}\right)^{1/2} \tag{F1}$$

where $C_v$ is the solids concentration by volume fraction, $d$ is median particle diameter, $R_h$ is the hydraulic radius of the naturally forming channel, with the densities referring to solid particle density and density of the carrier fluid. Through observation of natural channels, and sensitivity analysis of this model to channel shape, Fitton (2007) determined that predictions were generally insensitive to channel shape, and rather the aspect ratio between width and depth was the important parameter. Fitton (2007) recommends using a parabolic cross-section with a width to depth ratio of 5.5 for calculation of hydraulic radius.

For non-segregating slurries, Fitton (2007) proposed the following equation, based on experimental correlations of the critical velocity and the Reynolds number for a Bingham plastic observed in flume tests:

$$V_{c\ Non-seggregating} = 0.145ln\left(\frac{pVR_H}{\mu_{BP}}\right) \tag{F2}$$

Where the term in brackets is the Reynold's number for a Bingham plastic, ρ is bulk density of the slurry, $\mu_{BP}$ is the Bingham viscosity.

Based upon the assumed channel geometry, the slope may be determined using the following three equations, varying channel depth $d$ until $V=V_c$:

1. Assume a depth of channel, $d$
2. Calculate $V = Q/A$, where the channel is assumed to be a parabola with width =5.5 X $d$
3. Calculate $R_h$, and then compute $V_c$ using Equation 1 or 2. Repeat 1 through 3 until $V=V_c$.

4. Determine *Re*, using the definition of *Re* for a Herschel-Bulkley fluid, $Re_{HB} = \frac{8\rho V^2}{\tau_y+K\left(\frac{2V}{R_H}\right)^{\eta}}$

   The denominator of the equation is the Herschel-Bulkley equation for a flow curve, $K$ and $\eta$ are fitting parameters

5. *Re* is then used calculate the friction factor, *f*, using the Colebrook-White Equation:

$$\frac{1}{\sqrt{f}} = -2log\left[\frac{k_s}{14.8R_H} + \frac{2.51}{Re\sqrt{f}}\right] \tag{F3}$$

   In which $k_s$ is 2 X $D_{90}$.

6. The slope for uniform flow in the channel is calculated:

$$S_0=f\,V^2/(8gR_H) \tag{F4}$$

$S_o$ is taken to be the beach slope

## Comparisons with Field Observations and Experiments

The method was compared to a set of experiments conducted in a 10 m long flume, as well as an independent set of field data from ATC Williams. Details on the tailings used in the experiments are given in Table 1. The overall performance against both experiments and field data is given in Figure 3.

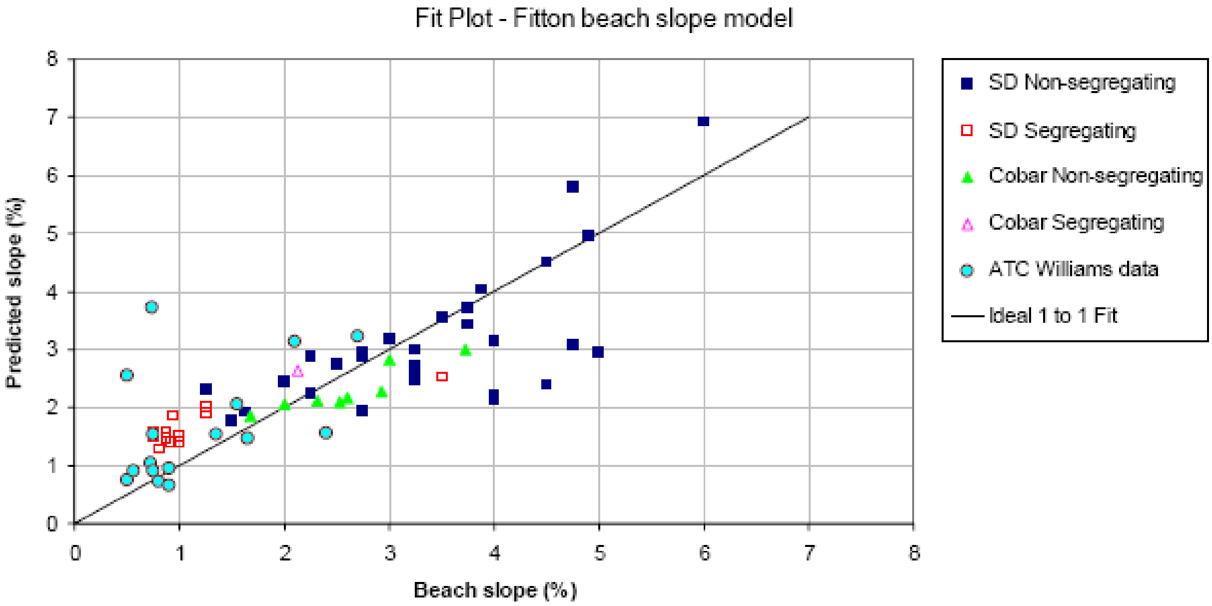



**Figure 3 Fitton model applied to data from large flume test and select field data (From Fitton 2007)**

| Experimental measurement | Cobar | Sunrise Dam |
|---|---|---|
| No. of equilibrium slopes recorded | 9 | 41 |
| Steepest equilibrium slope (%) | 3.7 | 6.0 |
| Flattest equilibrium slope (%) | 1.7 | 0.75 |
| Maximum flow rate (l/s) | 19 | 24 |
| Minimum flow rate (l/s) | 1.9 | 1.9 |
| Maximum concentration (% w/w) | 57.7 | 66.8 |
| Minimum concentration (% w/w) | 45.8 | 25.8 |
| Segregation threshold (% w/w) | ~ 46 | ~ 52 |
| Herschel-Bulkley rheological fit for slurry at maximum concentration: | | |
| Yield strength, $\tau_y$ (Pa) | 3.65 | 21.2 |
| Consistency index, $K$ (Pa.s) | 0.076 | 0.44 |
| Power, $n$ | 0.81 | 0.60 |
| Bingham plastic viscosity, $K_{BP}$ (Pa.s) | 0.016 | 0.030 |

**Table 1 Tailings used to generate experimental data set in Figure 3**

## Method 2 McPhail Stream Power Method

This method was initially proposed by McPhail (1995) and subsequently adapted to high density tailings (McPhail 2008). The basic premise of the method is that beach slope profile is related to the dissipation of energy of the tailings as they move downslope in a channel. The energy is quantified as "Stream Power" is this method:

$$P = \rho g Q H \qquad \text{(M1)}$$

In which H is the energy head. Pressure head is assumed to be small, and therefore, with respect to the datum of the tailings beach, Stream Power can be expressed in term of velocity head:

$$P_x = \rho Q g(\frac{v^2_x}{2g}) = \rho Q \frac{v^2_x}{2} \qquad \text{(M2)}$$

On a tailings beach, velocity head falls to zero at the toe. A infinite number of stream power functions may satisfy this criteria: McPhail (1995) derived a specific equation based on maximum information entropy technique, essentially a theory that states that the likeliest model for reality, is an equation in which its sensitivity to variable uncertainty is the highest possible, more details are given in McPhail (1995). The equation produced is:

$$P(x) = -\frac{1}{\beta} \ln\left[(1 - \exp^{-\beta P_0})\frac{x}{L} + \exp^{-\beta P_0}\right] \qquad \text{(M3)}$$

The parameter $P_0$ is the stream power at the start of the beach. *L* is the distance from the start of the beach to the toe. The parameter β is a constant emerging from the mathematical derivation of Equation (M3).

The slope at the start of the beach is obtained by differentiating Equation (3) with respect to x to obtain $S_0$ which yields the following equation:

$$S_0 = -\frac{(1 - \exp^{-\beta P_0})}{L\beta \exp^{-\beta P_0}} \qquad \text{(M4)}$$

The slope of the stream power curve, $S_B(x)$ and hence of the beach can be determined at any point, *x*, along the beach from the equation:

$$S_B(x) = -\frac{(1 - \exp^{-\beta P_0})}{L\beta \exp^{-\beta P(x)}} \qquad \text{(M5)}$$

The elevation, y, of the beach at any point, x, along the beach can be determined using the following equation and working in small increments of Δx from the start of the beach:

$$y = y(x + \Delta x) + S_B(x)\Delta x \qquad \text{(M6)}$$

Equation (M6) gives the beach profile.

The application of these equations requires the determination of i) the initial stream power of the tailings on the beach, and ii) the initial slope. McPhail (1998.1995) noted that the initial stream power is not the stream power of the tailings in the pipeline. The tailings will plunge from the pipe outlet, potentially undergoing a hydraulic jump to subcritical, and also potentially transforming back to supercritical flow in the formation of a channel. . The energy dissipation in the plunge pool may be calculated using the continuity equation in conjunction with the force-momentum flux equation and an assumption that the channel exiting the plunge pool will be semi-circular, the minimum flow section (McPhail 1998).

Small-scale beaching simulations can be used to estimate a realistic flow curve for segregating slurries. This information is then used to estimate the initial beach slope for full-scale predictions.

The application of this method to prediction of full-scale beaches may be accomplished by the following steps:

1. Perform beaching simulations at the anticipated range of slurry densities at a small scale. McPhail (1998) recommends depositing tailings in the corner of a paddock at least 8 m by 8 m to minimize wall effects. Fit the resulting beach profiles using Equation M3 to obtain β, $P_0$, and $S_0$.
2. Calculate velocity down the beach from Stream Power for known density and flow rate using Equation M2.
3. Construct a shear stress –shear rate relationships down the beach, for each slurry density, using the following equations. For shear rate:

   $$\omega = 2v / R_H \qquad \text{(M7)}$$

   where $R_H$ is the hydraulic radius of the flow stream at the lip of the plunge pool, and v is the mean velocity calculated in step 2. The shape of the flow channel is assumed to be semi-circular. Channel hydraulic radius follows from continuity for known Q (Given) and V (from step 2).

   The shear stress down the beach can be calculated using the following equation (Wasp et al, 1977):

   $$\tau = f_f \rho v^2 / 2 \qquad \text{(M8)}$$

   where $f_f$ is the Fanning friction coefficient (1/4 value of Darcy-Weisbach friction coefficient).
4. Use the full scale discharge data of slurry density, flow rate, pipe diameter, and residual pressure together with standard hydraulic calculations for the plunge pool to estimate the initial stream power for a given slurry density and flow rate for the field ($P_{of}$).
5. Calculate the initial velocity exiting the plunge pool using M2 from $P_{of}$.
6. Calculate the initial slope using:

   $$S_i = \tau / \rho g R_H \qquad \text{(M9)}$$

7. For a given length L, the field beach slope profile can then be estimated by defining β using M4, and subsequently using M3, M5, and M6 to plot the profile from the plunge pool to the supernatant pond.

## Comparisons with Field and Experimental data

One case from McPhail (1998) is presented. This case study entails a slurry of solids SG 3.6, a maximum particle size of 1.18 mm and a percentage passing 0.075 mm of 50%. The slurry was generated by hard rock mining.

Flume tests were carried out in a flume constructed directly adjacent to a pipe loop. At the end of a pipe loop test of a slurry at a particular density a portion of the flow stream was diverted to the flume. The flume measured 7 m long by 0.5 m wide. Discharge was through a 25mm pipe at a flow rate of 0.8 l/s such that the pipe was parallel to the beach and close to the beach elevation. The pipe was slowly raised as the beach built up. Figure 4 shows the measured beach profiles in the flume tests. Also indicated are fitted profiles based on the stream power equations described above.

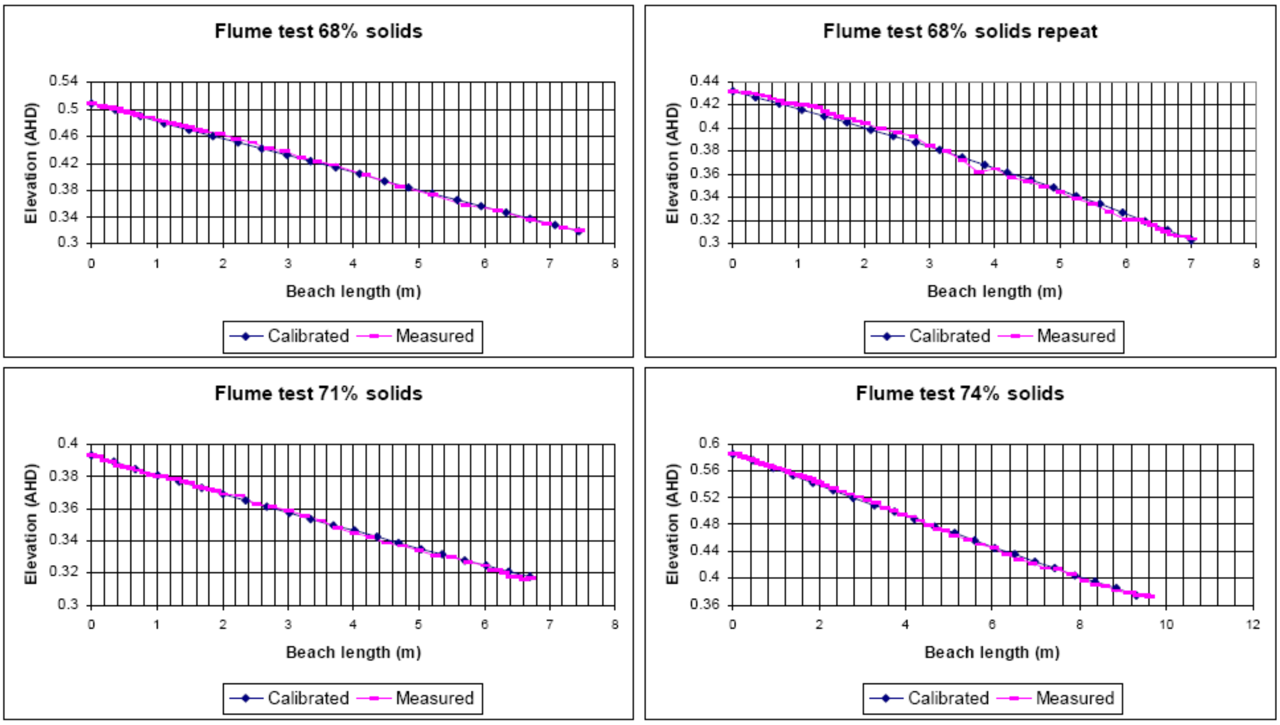



**Figure 4 Measured and fitted beach profiles for the flume tests**

Figure 5 shows measured beach profiles from the full scale field operations. These have been measured over the period from 2000 to 2004 and cover the pre-high density operations and three trial depositions all carried out in the course of routine operations as modifications to the thickening equipment were progressively implemented. Also indicated are the fitted profiles using the entropy-based stream power methodology as well as the calibrated stream power, shear rate, shear stress and fitted initial beach slope. It can be seen that the shape of the profiles are very well modelled by the method.

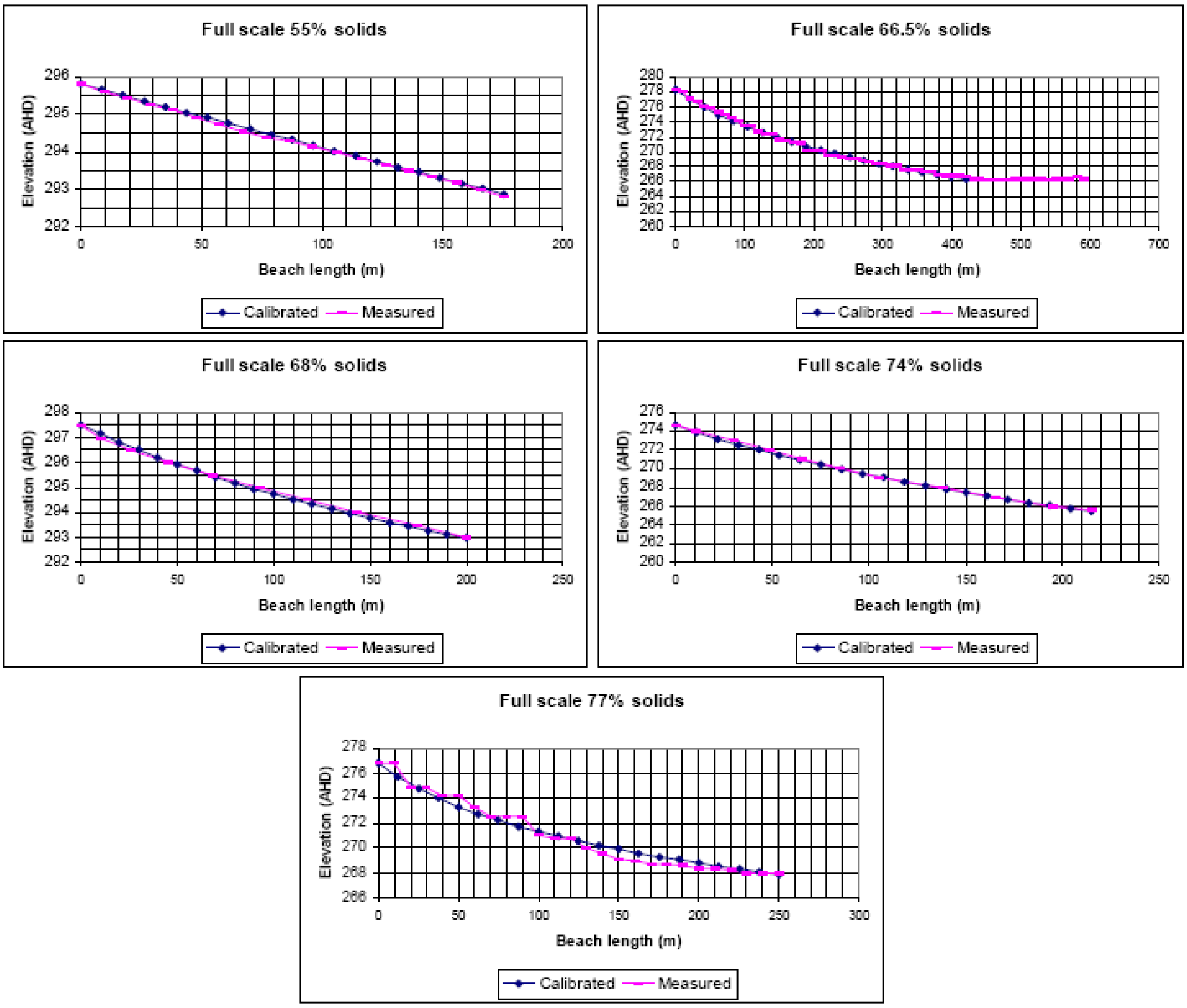



**Figure 5 Measured and fitted field beach slope profiles**

## Method 3: Lubrication theory equations

Lubrication theory (LT) allows for reduction of the Navier-Stokes Equations to relatively simple equations for equilibrium profiles of simple geometries for non-Newtonian fluids. The reduction is afforded by assuming:

1. Flows have a small aspect ratio (thickness / length ratio is small)
2. The ratio of inertial forces to gravitational and viscous forces are small. Therefore, LT theories are valid for a non-Newtonian fluid that spreads under its own weight, but the flow rate out of the pipe does not enter the equations.

These simplified momentum and continuity equations have been solved analytically for yield stress fluids under special geometries and special conditions by several researchers, for applications such as mud or lava flow (Yuhi and Mei, 2004; Liu and Mei, 1989; Balmforth et al., 2002; Coussot and Proust, 1996). Considering that on an inclined bed the flow is not driven by any external pressure, but rather by the volume body force of gravity, then the momentum equation in the direction of the flow is as below (Liu and Mei, 1989):

$$\rho g \sin\theta - \frac{\partial p}{\partial x} + \frac{\partial \tau}{\partial z} = 0 \quad \text{(L1)}$$

where p is pressure, ρ is density, h is the angle of the inclined surface from the horizontal, g is the acceleration due to gravity, τ is the shear stress, the x-axis is the direction of the inclined plane, and the z-axis is perpendicular to the inclined plane. If one further assumes the pressure distribution to be hydrostatic, then:

$$p = \rho g (h - z)\cos\theta \quad \text{(L2)}$$

Where h is the depth of the free surface of the flow. Then differentiating (2) to substitute it into the left-hand side of (1), and solving for the shear stress, one obtains an expression in terms of depth z:

$$\tau = \rho g (h - z)\cos\theta(\tan\theta - \frac{\partial h}{\partial x}) \quad \text{(L3)}$$

Now, setting z = 0, one may obtain an expression for the steady-state profile of a Bingham fluid, given the condition τ< τy, the yield stress, The equation for a flat bed is:

$$\tau_y (x - x_0) = \frac{\rho g}{2}(h^2 - h_0^2) \quad \text{(L4)}$$

where *h0* is the height at *x0*. In the same vein as the derivation of Eq. (4), an equation for the steady-state profile of flow from the top of an inclined hill may be derived (Yuhi and Mei, 2004):

$$h' - h_0' + \ln(1 - h') = x' - x_0' \quad \text{(L5)}$$

where h' and x' are normalized dimensionless variables, such that *h = h' [ τy/(ρgsinθ) ]* and *x = x'cotθ [ τy/(ρgsinθ)* ].

As shown in Simms (2007), Henriquez and Simms (2009), and Mizani et al. (2010a and b), these equations show good skill in fitting equilibrium flume profiles for both single and multilayer deposits at the laboratory scale ( <3m), for gold tailings and kaolinite; and have been fit to single layer profiles at a gold mine (~ 50 m run out). An important aspect of these equations is they shown the scale dependency of the overall angle of single layer flows (Figure 6). Therefore, the overall angle observed in flume tests cannot be directly extrapolated to the field. In the following figures, L4 is used to model the first layer, whereas L5 is used to fit all subsequent layers, using the average angle for the previous layer. In all the tests shown, each layer was left for 24 hours before the next layer is placed. This generally coincided with the end of settling but before desiccation. Figure 7 shows the fit to a 3-D small scale simulation of stacking: for each layer, the same density and yield stress (1900 kg/m$^3$ and 30 Pa) are used.

The LT theory works well up to a point. As illustrated in Figure 8, once a deposit reaches a certain slope, there is a change in flow regime and the tailings begin to flow is channels from the deposition point some distance down the slope. This is coincident with the beginning of concavity in the beach slope profile, and failure of the LT equations

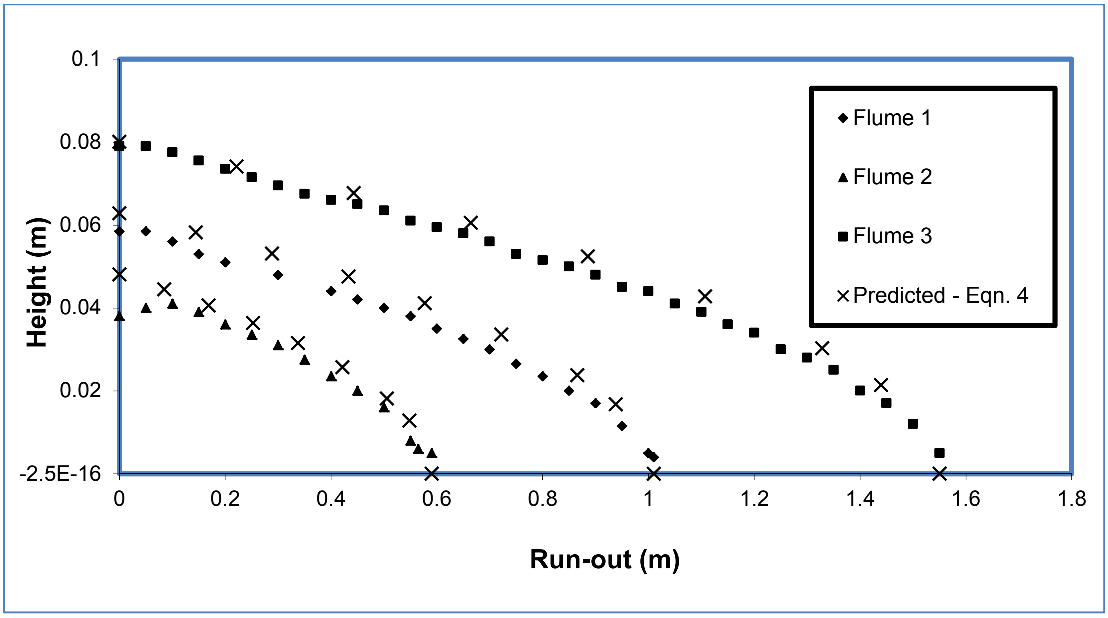



**Figure 6 Three single layer flows of different volume, each fit with Equation 4 using the same material parameters (Density and yield stress)**

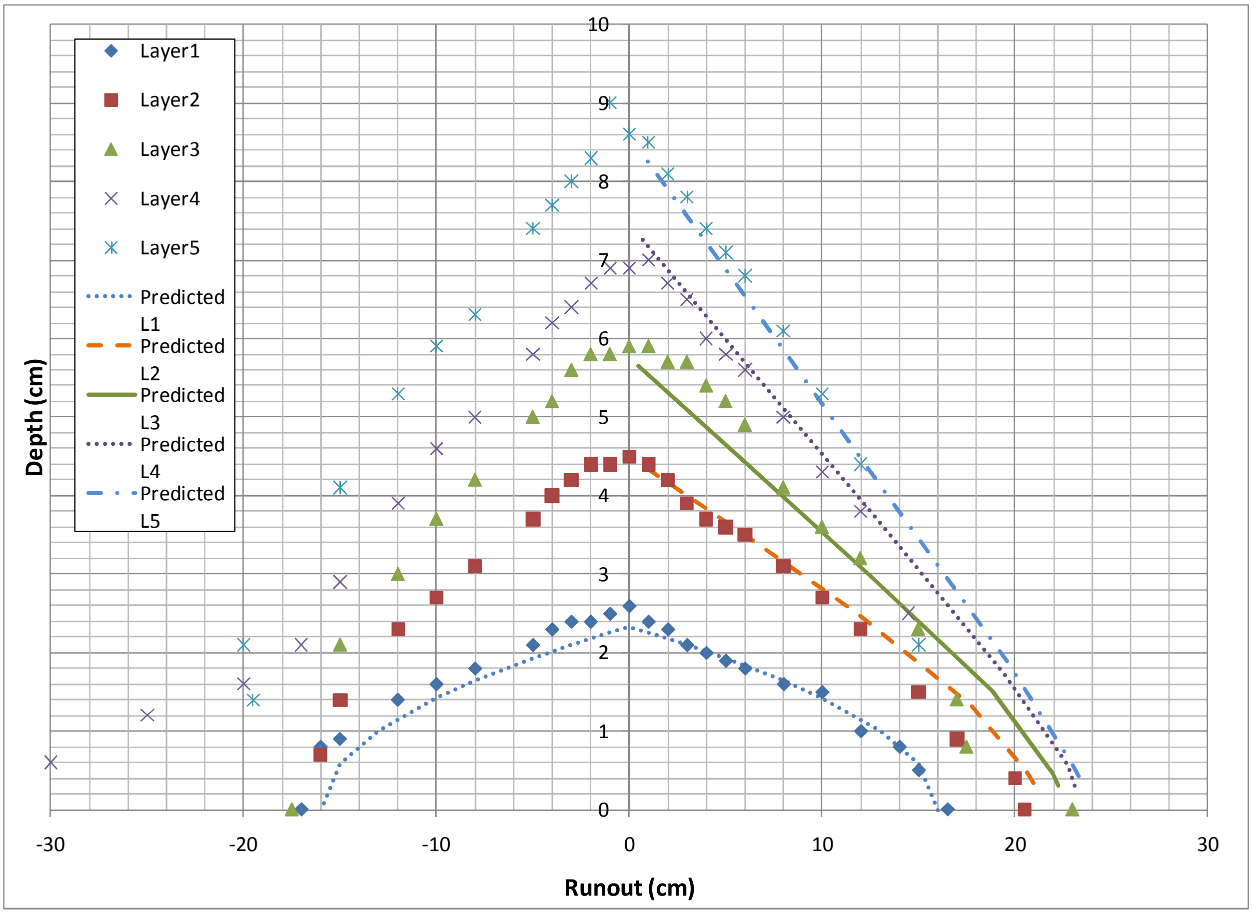
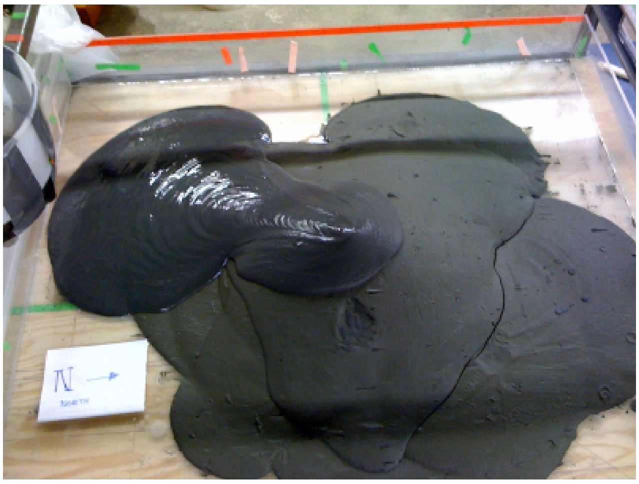



**Figure 7 Early deposition in a small-scale 3D simulation, Q=0.4 LPM, 2.6 L per layer, Yield stress 30 Pa, Density 1900 kg/m$^3$**



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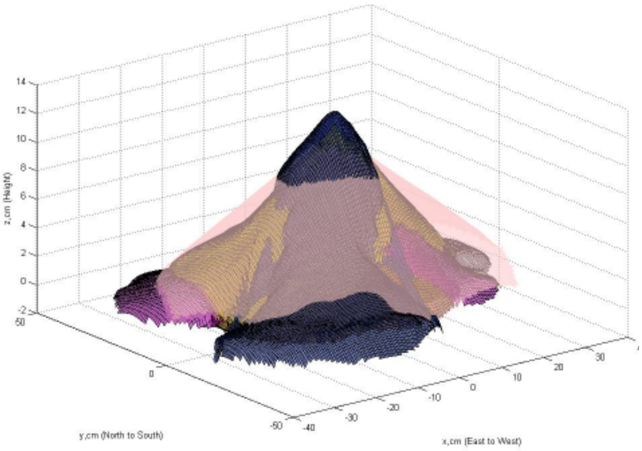

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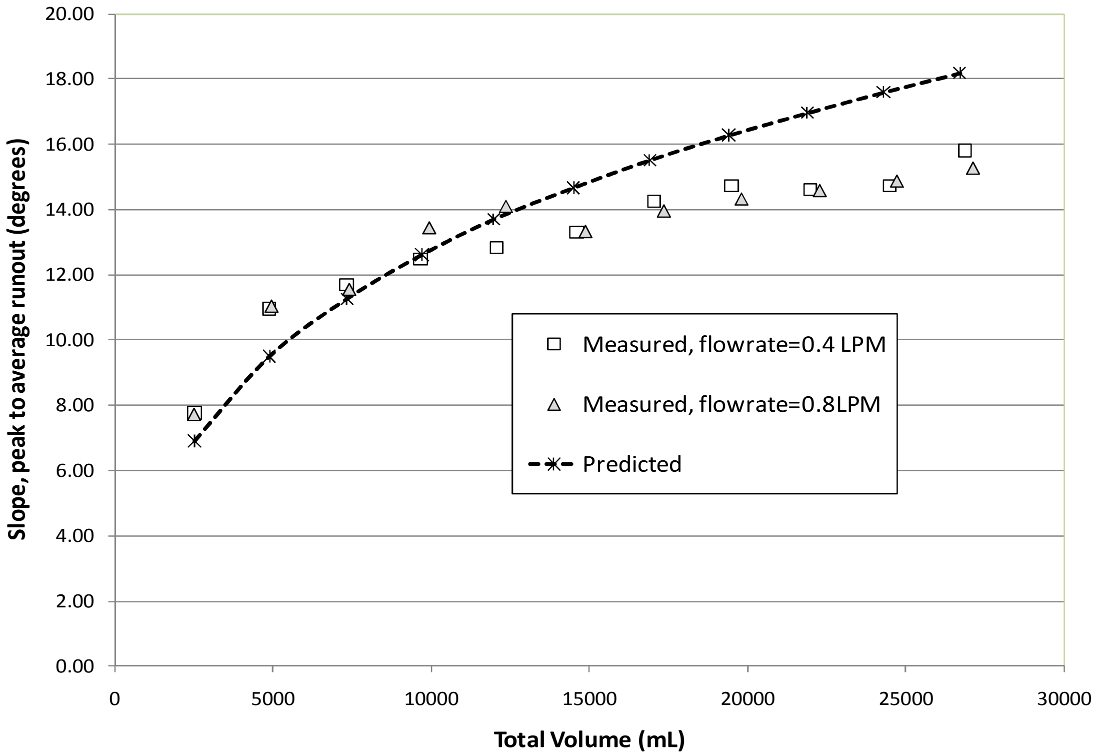



(c)

**Figure 8 Change in flow regime from "spreading" to channelized flow, coincident with increasing concavity of slope profile. Each point denotes a layer, 2.6 litres each. (a) and (b) show a deposit with 8 layers, pink cone in (b) is the predicted shape using the LT equations**

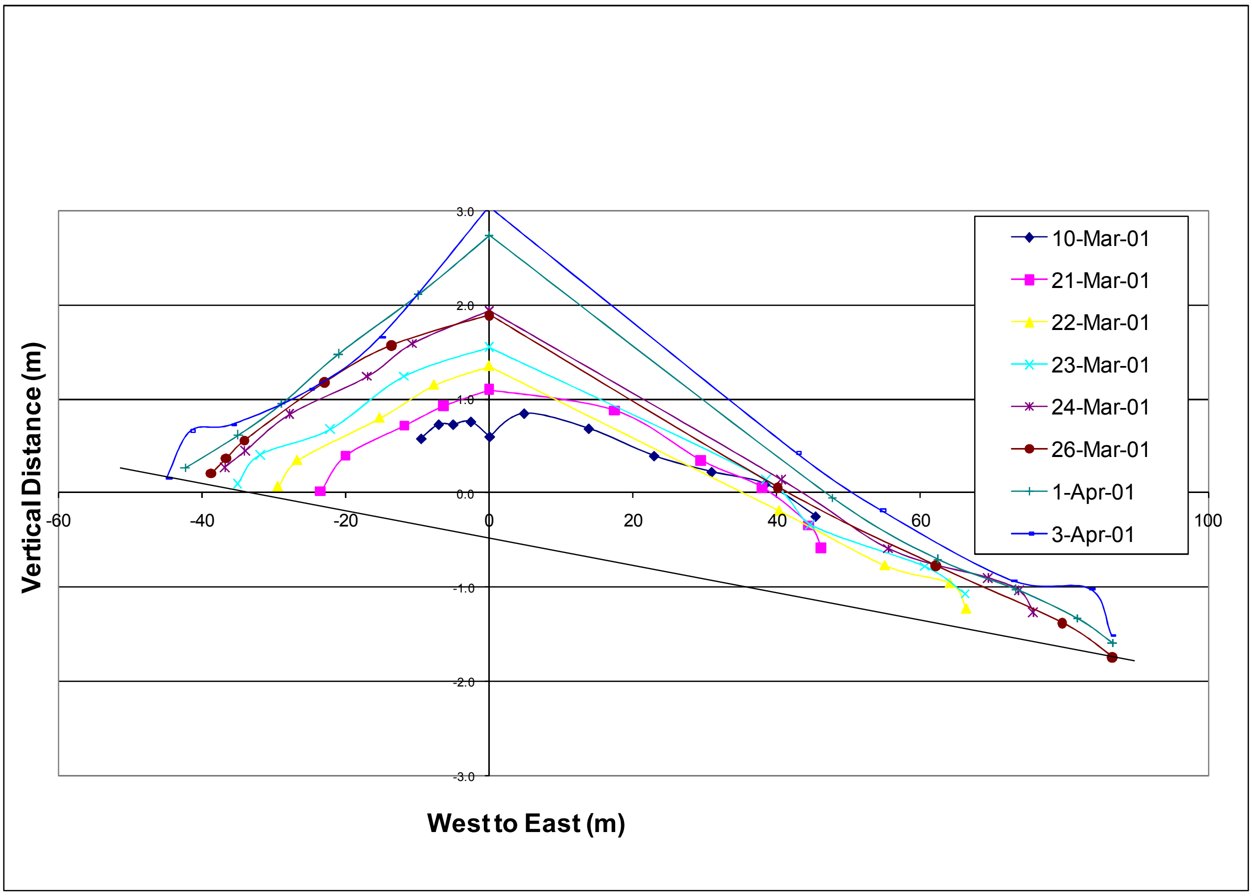



**Figure 9 Early deposition at one tower at Bulyanhulu (Crowder 2007)**

The same trend seen in Figures 7 and 8 can be observed during early deposition at the Bulyanhulu site (Figure 9). The shape of the tailings stack evolves from a convex profile, becoming more linear and eventually concave. This progression of slope shapes is observed at other thickened tailings sites, as noted by Williams and Meynink (1986) and Fitton (2007).

It is of interest to define the slope when the flow changes from spreading to channel flow. This is not only of interest as to the limitation of the LT approach, but also tells the engineer when the flow becomes less manageable, with respect to controlling individual lift thickness. One hypothesis, as yet unproven, is that limits of the LT equations can be defined by a Froude number. One can approximately find the Froude number as a function of distance from the deposition point, *r*, assuming uniform axisymmetric flow down a uniform slope, and using Manning's equation and continuity to derives expressions for depth *d* and mean velocity as a function of distance from the deposition point, *r*:

$$d = \left(\frac{Qn}{2\pi r {S_0}^{1/2}}\right)^{3/5} \tag{L6}$$

$$V = \frac{Q}{2\pi r d} \tag{L7}$$

Where *n* is Manning's coefficient for overland flow. The Froude number, $V/(gd)^{0.5}$, can then be evaluated as a function of radial distance from the deposition point.

A solution is shown in Figure 10, for *Q*=0.0205 and *n*= 0.06 for two slopes. It is seen that the Froude number decreases down the beach. Thus, in order to evaluate the stability of spreading flow, a realistic minimum radius of flow must be selected. Selecting the minimum radius as the radius of the pipe seems a reasonable and probably conservative assumption, as increasing *r* would increase the critical slope.

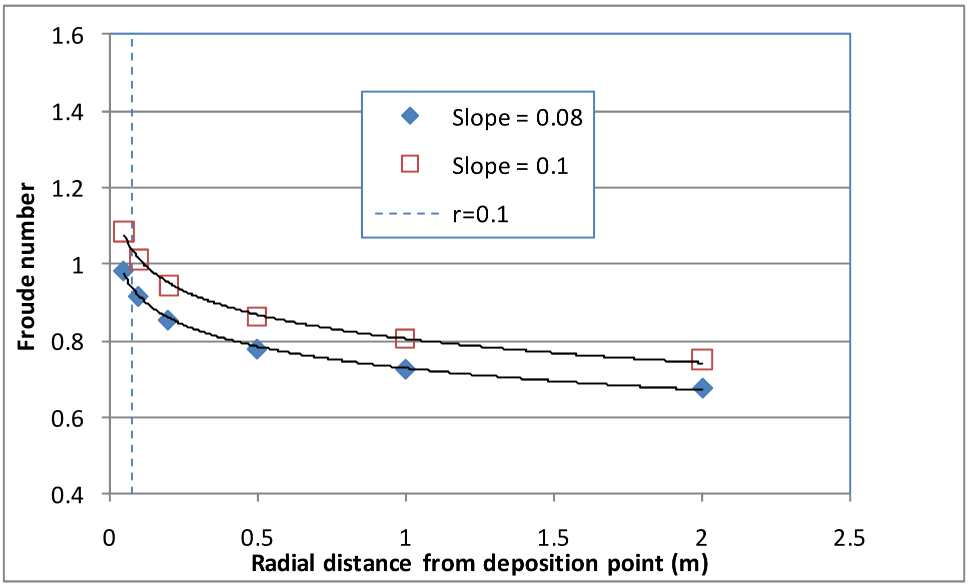
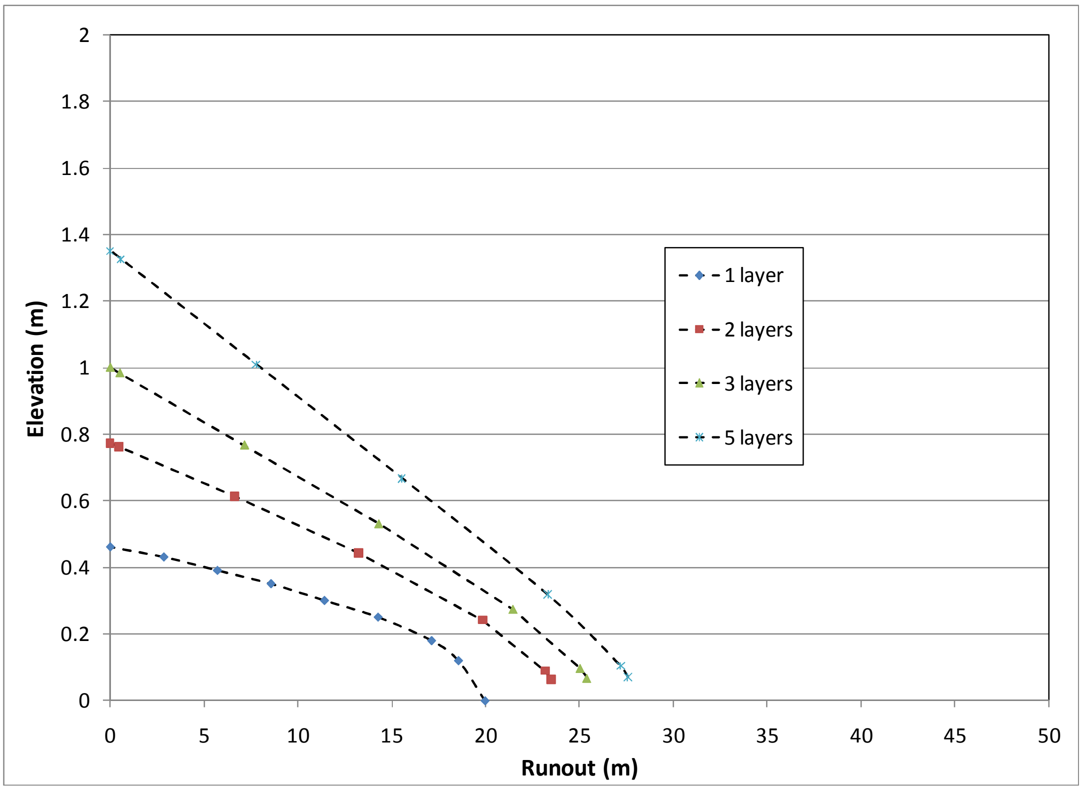



**Figure 10 Simple estimate of Froude number down the beach, for initially axisymmetric spreading flow. Q=0.0205 $m^3/s$, n=0.6**

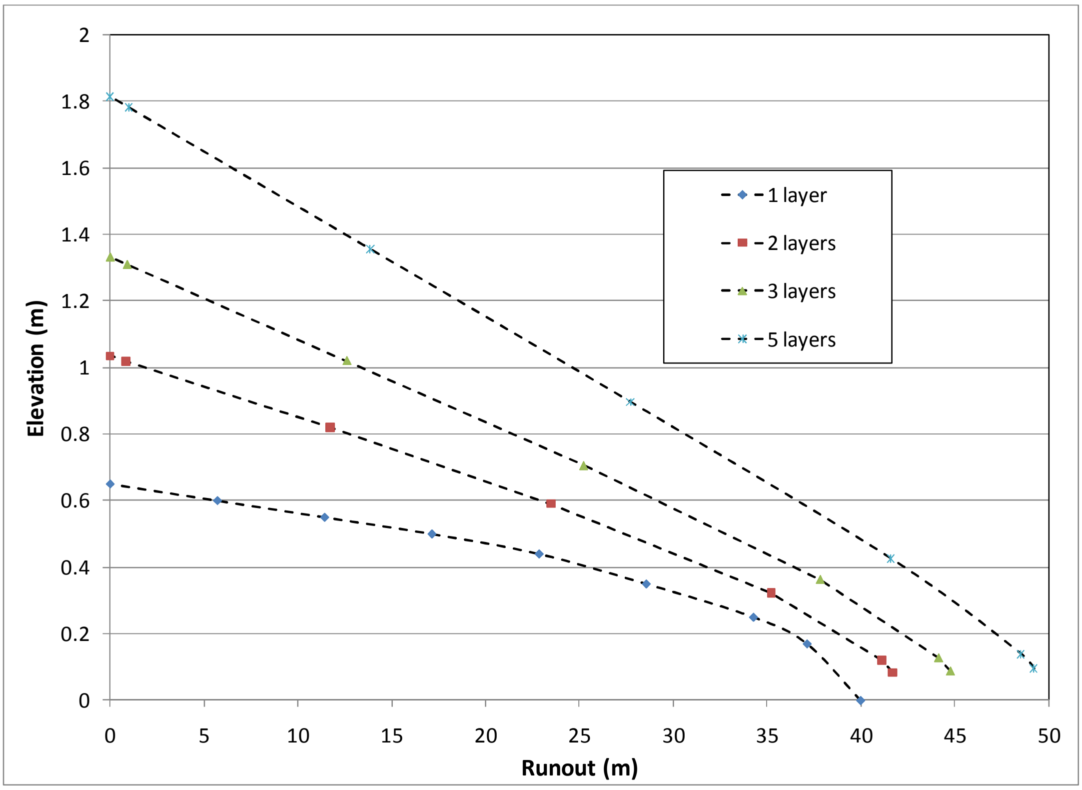
In this example, flow rate corresponds to that of the Bulyanhulu Mine, for early deposition, such as presented in Figure 10. The Manning' s coefficient was determined by applying the L6 and L7 to the small-scale deposition simulations in Figures 7 and 8, when the critical angle is known from Figure 8c, and the onset of channelling flow was assumed to correspond to Fr=1. A slope of 0.10 gives a Froude number of ~ 1 for a pipe radius of 0.1 m. Looking at the slope of Figure 9 with relation to the horizontal, we can approximate the slope when a concave beach slope starts to develop at the data recorded on April 1$^{st}$, on the West side of the deposit. The elevation is ~2.8 m, and the runout ~ 42 m, gives a slope of 0.067. This is promising but likely more refinement of L6 and L7 is probable in the face of other data from other sites.

An important note is that the even “non-segregating” tailings may exhibit significant thixotropic behaviour while they are flowing (Mizani et al 2010 a and b). This occurs both by natural settling and by adsorption of moisture by the underlying tailings. Thus the best-fit yield stress for the LT equations for small-flume tests may underestimate the value relevant to the field (30 Pa compared to 100 Pa for the Bulyanhulu tailings). Mizani et al. (2010b) recommends using slow rates of deposition in multilayer trial tests with dimensions as large as practicable to simulate the same time of deposition as would occur in the field, where the underlying tailings are left to dry significantly before placement of the next layer.

Finally, given the restriction of the LT approach to spreading flow, what is its utility? One, it can be used to estimate or model layer thickness as the stack initially builds up. Second, it can be used to model how quickly the stack will arrive at the critical angle. This is of interest to engineers who wish to control layer thickness in a cyclic deposition scheme, as beyond the onset of channelling flow control of layer thickness is more challenging. . Consider Figure 11, where two alternate deposition schemes are contemplated. It can be seen that the deposition scheme with the longer cycle time, allows for more volume to be deposited at a given deposition point, before the critical angle is reached.




(a)




(b)

**Figure 11 Improving deposition point capacity by increasing deposition volume below critical angle, as per Method 3 equations**

## Discussion: Comparison of Methods

To illustrate the relations between the different methods, they will be compared to evolution of stack geometry at the Bulyanhulu mine. While this mine is not typical of thickened tailings deposits, due to the relatively high solids concentration and cyclic deposition scheme, the relatively rich data set on field profiles and rheological properties that is available in the literature allows for some interesting comparisons. Early deposition (Figure 9) shows gradually increasing overall angles, which achieve a maximum slope between 1:10 and 1:14 or 10% and 7% (Shuttleworth 2005). Later observations (Addis and Cunningham 2010) showed the development of a highly concave profile, the beach slope ranging from 9% within 30 m of the deposition point, and a 3% slope fit through the peak and the toes for run outs of ~ 300m. The overall slope, in terms of estimating a cone of equal volume for the footprint, was ~ 4%.

This history is interesting as it suggests that the overall slope achieved a maximum value, and that the overall slope begins to decrease once a concave slope profile is established. This is of interest to mine operators in terms of maximizing footprint capacity and for efficient spreading of the tailings.

Figure 12 presents a comparison of predicted beach slope profiles using Methods 1 and 2 with the beach slope profile measured by Addis and Cunningham (2010). The data required for the methods were obtained by laboratory measured parameters presented in Kwak et al. (2005) and Henriquez and Simms (2009). A Bingham model was assumed for the tailing's rheogram, with a Bingham viscosity of 0.1 PaS measured by Kwak et al. (2005). Two yield stresses are used in the predictions, one value of 50 Pa measured by Kwak et al. (2005), one value of 100 Pa shown to be a best fit the individual layer profiles in the field (Henriquez and Simms 2009). Mizani et al. (2010) argued that 100 Pa may be more applicable to the field due to thixotropic effects manifested atr the field scale. Other data used in the predictions include $D_{90}$ (80 microns), slurry density (1900 kg/m$^3$), and flow rate (20.5 l/s).

For Method 1, Equation F2, the critical velocity for non-settling slurry, is employed. The prediction employing the higher yield stress gives a very good estimate of the overall slope (4%), while the prediction employing the lower yield stress is 2.7%. For Method 2, the dissipation of the energy in the plunge pool is ignored due to lack of data on tower height. The best fit with the field data is for a yield stress of 50 Pa. Yield stresses factor into the estimate of initial slope, therefore a higher yield stress will increase the concavity of the predicted profile for a fixed length estimates this shape. <u>As this is a prediction of a single profile, it cannot be seen to validate either method, due to the danger of a fortuitous result.</u> However, what is of interest is the sensitivity in the predictions to a variation in the parameters, in this case, the yield stress. In both cases, an approximately 1.5% variation in average slope occurs due the variation in yield stress. All authors stress the importance of estimate the relevant field properties, especially with respect to rheology.

On the issue of concavity, the authors propose two explanations why non-segregating tailings should produce an equilibrium beach slope profile with curvature. Fitton (2007) ascribes concavity to variability in the properties of the tailings delivered, and has developed beach slope modelling methods using this assumption (Fitton et al. 2008). By contrast, in the method of McPhail (1998) concavity is built into the equation for stream power dissipation. Additionally, the experiments presented in the description of Method 3 show that concavity seems to arise under controlled conditions at fixed flow rates and slurry concentrations, at a repeatable angle for a given tailings deposited at a given flow rate, and is coincident with the initiation of channel flow. One of the authors notes that concavity is seen in natural topographies where significant rain erosion occurs, indeed, soil covers are sometimes designed to mimic these natural topographies to minimize damage to the cover by rain erosion (Ayres et al 2007). These possible explanations of concavity are not mutually exclusive.

Given the inherent uncertainty associated with prediction of beach slope profiles, it is better for engineers to manage this uncertainty. All three approaches show that the average equilibrium slope is a function of flow rate, and that slower flow rates produce steeper angles. Therefore, practitioners may wish to explore the simultaneous use of multiple deposition points or spigots, to minimize tailings channel velocity and subsequent erosion.

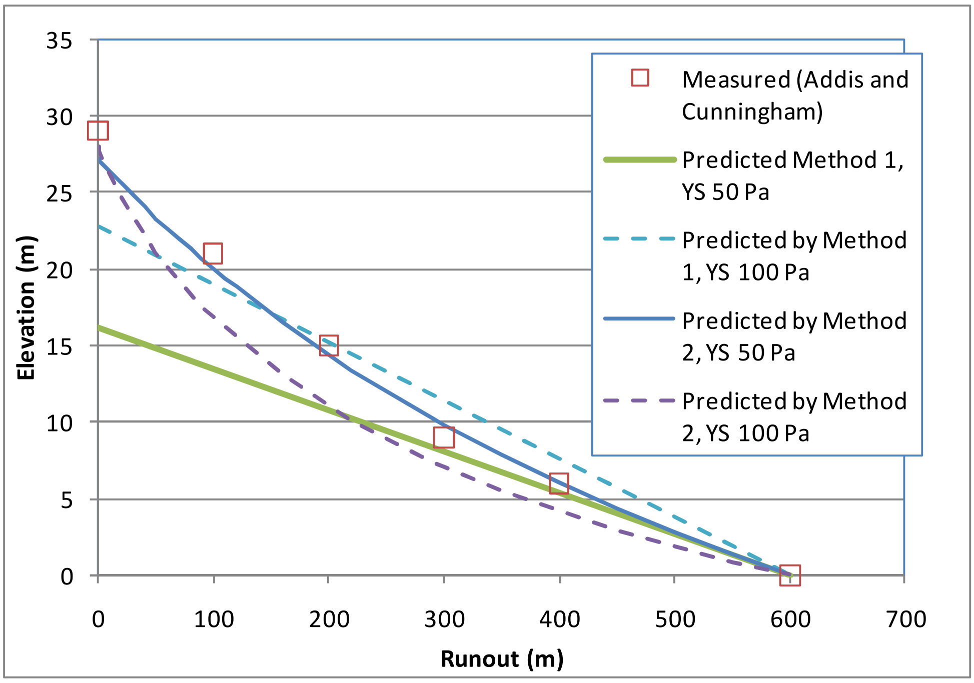



**Figure 12 Predictions of beach slope profile for data on Bulyanhulu from Addis and Cunningham (2010), for assumed yield stresses of 50 and 100 Pa**

## Summary and Conclusions

Three methods of stack profile prediction are examined. These methods, despite different philosophies and contexts in which they were developed, complement each other to some degree, and each may be useful to engineers to anticipate stack geometry evolution and to predict thickened tailings beach slopes. Methods 1 and 2 are based on analysis of channelized flows that develop later in tailings deposition. Method 1 assumes the equilibrium profile is reached when erosion and sedimentation are in equilibrium, and determines this point through uniform flow calculations, assuming a parabolic channel cross-section. Method 2 is based on dissipation of energy down slope, and is implicitly a non-uniform flow analysis. Method 3 uses lubrication theory to predict equilibrium profiles of spreading flow for tailings of constant material properties.

Each method was applied to analyze the evolution of geometry at the Bulyanhulu site, a non-segregating relatively highly thickened gold tailings, which is characterized by the overall beach slope reaching a maximum value in early deposition, after which the overall slope degrades with time. It is suggested that this degradation is coincident with the initiation of a concavity in the profile.

Method 1 and Method 2 both gave reasonable estimates of the equilibrium profile at Bulyanhulu, for a plausible set of material parameters. However, the sensitivity of the both method's predictions to the plausible range of yield stress, illustrates the importance of determining the rheological properties relevant to the field, which may not correspond to laboratory rheometry tests. All of the authors recommend various kinds of pilot-scale tests to investigate field scale behaviour, where practicable.

Whereas Method 1 is the most straightforward method to predict equilibrium beach slope, Method 2 is the only method that directly predicts a concave profile. Method 3 may find application in estimating layer thickness and beach slope evolution of cyclic deposition schemes during the early evolution of the impoundment. A calculation based on estimating the onset of channel flow is proposed to define the upper limit of this method. This limit may also prove to be useful at operations where control of individual layer geometry is desired for purposes of regulating desiccation.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